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THE GROUND-WAVE ATTENUATION FUNCTION FOR PROPAGATION OVER A HIGHLY INDUCTIVE SURFACE

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Contract No. PRO-65-504

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Scientific Report No. 37

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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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ABSTRACT

Propagation of an electromagnetic ground wave over a plane surface in which the argument of the surface impedance is greater than $\pi/4$ but less than $\pi/2$ is considered in some detail. The numerical distance, p, over such a surface is characterized by $0 \lesssim \arg p \lesssim \pi/2$. The ground wave behaves in a rather unusual manner, and this is attributed to the interaction of phasors representing a trapped wave and a Norton surface wave. Approximate expressions are derived which determine the magnitude of the ground wave attenuation function at its maxima and minima as well as the phase at these points and the numerical distances where these maxima and minima occur. A method is also given for estimating the asymptotic phase for iarge $|p|$ which was previously not possible. Finally, detailed curves are presented which show the amplitude and phase of the ground wave attenuation function versus p. These curves should prove useful to practicing radio engineers attempting to make calculations for surface wave propagation over corrugated, stratified or rough surfaces.

THE GROUND-WAVE ATTENUATION FUNCTION FOR PROPAGATION OVER A HIGHLY INDUCTIVE SURFACE

R. J. King and G. A. Schlak

1. Introduction

The calculation of the field intensity of an electromagnetic ground wave propagating over a flat surface has received considerable attention in the past five decades. It is not the intent of this paper to dwell upon the many noteworthy contributions to this phenomenon or even to improve or extend them. Rather, an attempt has been made to explain some of the unusual aspects of propagation over a highly inductive surface. Having first done this, a comprehensive set of curves will be presented showing the amplitude and phase of the ground wave field which should prove useful to radio engineers applying surface wave propagation to corrugated, stratified or rough surfaces.

In the generally accepted context, a passive inductive surface is characterized by a surface impedance $Z_s = |Z_s|e^{i\phi_s}$ for which the phase ϕ_s lies in the range $0 \leq \phi_s \leq \pi/2$. The time excitation factor is assumed to be exp $[i\omega t]$, and Z_s is defined as the ratio of the tangential electric field to the tangential magnetic field at the surface. For an arbitrary passive surface, $-\pi/2 \leq \phi_s \leq \pi/2$, and for any homogeneous surface, $0 \leq \phi_s < \pi/4$; the lower value corresponds to a perfect dielectric and the upper value corresponds to a perfect conductor. In the sense used here, a highly inductive passive surface implies that $\pi/4 \leq \phi_s \leq \pi/2$, and it is within this range of ϕ_s where some unusual effects are noted. Although the following applies equally well to a general surface in which $-\pi/2 \leq \phi_s \leq \pi/2$, this paper is primarily concerned with a highly inductive surface defined above.

Propagation over a highly inductive surface is not merely an academic problem. For example, ϕ_s can lie in the range $\pi/4 \leq \phi_s \leq \pi/2$ when propagating over a horizontally stratified medium [Attwood, 1951; Barlow and Cullen, 1953; Wait, 1953, 1957, 1962a, b, 1964; Fernando and Barlow, 1956], a corrugated surface [Barlow and Karbowiak, 1954; Zucker, 1954; Fernando and Barlow, 1956; Wait, 1957], or a surface which is uniformly rough [Wait, 1959]. Approximations will be stated wherever appropriate, but the reader should be certain that other assumptions besides those which apply to the general problem are not violated. In particular, he should be cautious in applying the surface impedance concept and any approximations used in equations which describe Z_s for a particular surface.

Following Wait's [Wait, 1963, 1965b] suggestion, the type of interface wave excited by a dipole antenna over a conducting homogeneous half-space is commonly called the "Norton Surface Wave," obviously being named after its proponent. In addition, a second type of surface wave - the "trapped wave" or "Barlow wave" - propagates when $\pi/4 \leq \phi_s \leq \pi/2$ (e.g., when the lower half-space is not homogeneous). The combination of these waves has proven particularly useful at low and very low frequencies (3-300 KHz) where it is often the primary means of communication and navigation over the surface of the earth or ground. In this paper, it is this total wave which is denoted as the "ground wave." Schelkunoff [1959] has also listed the various varieties of surface waves and their distinguishing physical properties.

Excellent common examples of LF and VLF propagation over stratified media are arctic ice on a nearly perfectly conducting sea or a poorly conducting layer of earth lying above a highly conducting sub-stratum. Indeed, some of the observed experimental anomalies over a supposedly plane, homogeneous earth can be explained by assuming that the ground is stratified or rough. It should be emphasized however, that little experimental evidence presently exists which verifies the theory of propagation over a reactive surface, particularly an earth which is highly inductive. Experimental studies using models at microwave frequencies are currently being conducted at the University of Colorado in an attempt to confirm the theory, and this data will be available in the near future. Biggs and Swarm [1965] have conducted an analytic feasibility study of placing a horizontal electric dipole in Antarctic terrain, and construction of this antenna is now nearly complete.

Fortunately, the most important components of the electromagnetic fields of elementary antennas, i.e., short electric and magnetic dipoles which are either vertical or horizontal to the surface, can be expressed in terms of the now famous "Sommerfeld ground-wave attenuation function" [Sommerfeld, 1926]. Although this function was derived assuming a plane, homogeneous earth, Wait [1957, 1962a, 1962b, 1964] has shown that it applies to a stratified and conducting isotropic plane earth for which $|Z_s|^2 << \eta_o^2$, provided the strata are sufficiently lossy to avoid exciting waveguide modes. $\eta_o \cong 120\pi$ is the intrinsic impedance of free space. The validity of this is most easily seen by employing

the surface impedance concept which is particularly useful in problems of this sort. Godziński [1961] has shown that once the surface impedance is known, the <u>effective</u> electrical parameters (i.e., permittivity, permeability, and conductivity, or equivalently, the complex index of refraction) of a corresponding homogeneous ground can be found.

The requirement that the surface impedance be known is not complicating the problem because the description of all non-homogeneous surfaces (stratified, corrugated, rough, etc.) requires some involved, frequency-sensitive expression, and the surface impedance has been selected here. In essence, one need only calculate the surface impedance for a specific passive surface in order to know how the ground wave will behave.

Of course, the following analysis only applies in those cases where the surface impedance concept is valid in the solution of the corresponding boundary value problem. For example, it would not apply to the previously mentioned situation of a stratified earth when one of the layers is capable of supporting a waveguide mode. The essential requirement is that the local tangential fields be nearly plane [Godziński, 1961; Wait and Pope, 1954; Wait, 1961]. For an actual earth in which the magnitude of the effective complex index of refraction is much less than unity, the region where this is violated is a very small part of the important region, and can therefore be regarded as insignificant. This is especially true when the fields in the far zone are being considered. In terms of Z_s, it is sufficient to require that $|Z_s|^2 << \eta_o^2$. Incidentally, this is equivalent to the same condition imposed by Sommerfeld [1909, 1926] and by Norton [1937] in

deriving the attenuation function for propagation over a homogeneous earth.

Insofar as the character of the source is concerned, Norton [1937] gives completely general formulas(valid for distances greater than λ, the free space wavelength)for the vector electric field at any point above the surface of a plane earth and a radiating system which may consist of any configuration of vertical and horizontal electric dipoles. Formulas are also given for small loop antennas (magnetic dipoles) with their axes parallel or perpendicular to the earth. Similar formulas have been compiled by R. W. P. King [1956]. Wait [1961] has derived overlapping formulas for distance ranges extending from the near to the far zones, assuming the source is a horizontal electric dipole which is over, on, or below the surface of a homogeneous half-space. Biggs and Swarm [1965] have treated the far-zone fields of an inclined electric dipole on or below the surface of a homogeneous or horizontally stratified media. In most cases the vertical component of the electric field in the far zone, with which this paper is primarily concerned, is expressed in terms of the <u>same</u> ground wave attenuation function. Hence, only the results for a short vertical electric dipole will be shown and the reader is referred to the literature when he is concerned with the form of the electric field for a different type of antenna.

Secondary fields arising via reflections from other than the surface itself are to be ignored, e.g., ionospheric reflections, tropospheric scattering and obstacles other than those used in the theory of rough surfaces.

2. Theory

The transmitting antenna is assumed to be a short vertical electric dipole located at height h above the plane surface defined by $z = 0$. The vertical electric field at P which is at height z above the air-surface interface is approximately [Norton, 1937]

$$E_z(P) = \frac{-ik\eta_o I\ell}{4\pi}\left[\cos^2\psi_d \frac{e^{-ikD}}{D} + R_v \cos^2\psi \frac{e^{-ikR}}{R} + (1-R_v)F(w)(1-\Delta^2)\frac{e^{-ikR}}{R}\right], \tag{1}$$

where $k = 2\pi/\lambda$, I is the dipole current and ℓ is its length ($\ell << \lambda$). The "line of sight" distance from the source to the observation point P is D, and R is the distance from the image of the source to P. $\psi_d = \sin^{-1}(z-h)/D$ and $\psi = \sin^{-1}(z + h)/R$. $R_v = (S-\Delta)/(S+\Delta)$ is the reflection coefficient where $S = \sin\psi$ and $\Delta = Z_s/\eta_o$. Z_s is the surface impedance relating the tangential electric and magnetic fields of the source at the plane air-ground interface and, in general, is a function of ψ. For a homogeneous earth, R_v is equivalent to the Fresnel reflection coefficient for parallel polarization. The expression for $E_z(P)$ in terms of a surface impedance rather than the electrical constants of the earth was introduced by Wait [1962a] who pointed out the vastly extended usefulness of existing theory with this general approach.

F(w) is Sommerfeld's [1926] renowned "attenuation function" defined by

$$F(w) = 1 - (\pi w)^{1/2} e^{-w} \operatorname{erfc}[i(w)^{1/2}] \tag{2}$$

where

$$w = p\,(1 + S/\Delta)^2 = |w|\, e^{iB} \tag{3}$$

$$p = \frac{-ikR}{2} \Delta^2 = |p|e^{ib}, \tag{4}$$

and

$$b = 2\Phi_s - \pi/2. \tag{5}$$

The first term on the right hand side of (1) is the "direct" wave which exists with or without the presence of the ground. The second term is the specularly reflected wave which can be regarded as a wave originating from an image of the source located at $z = -h$. The combination of these two waves is commonly called the "space" wave. The third term, which is of primary interest here, has been defined as the surface wave which is significant only close to the air-surface interface. Note that (1) leads to all of the limiting forms one would intuitively expect. For example, if $h = z = o$, $R_v = -1$ and the space wave vanishes leaving only the surface wave. On the other hand, if $Z_s = o$ (perfectly conducting ground) then $R_v = +1$ and the surface wave vanishes. Equations (1) through (5) apply to an arbitrary ground for which $-\pi/2 \leq \Phi_s \leq \pi/2$ provided $|\Delta|^2 << 1$. An additional restriction required in (1) and (2) is that $k\rho >> 1$ and $\rho >> (z + h)$ where $\rho = D \cos \psi_d$ is the horizontal separation distance between the source and the observation point P. Thus, the Norton solution for the homogeneous earth can be considered as a special case of the more general surface impedance formulation.

The dimensionless parameter p given by (4) fits so nicely into the definition of F(p) that Sommerfeld [1926] assigned the name "numerical distance" to p. This terminology has been widely accepted, and it has become common practice to discuss ground waves in terms

of the numerical distance, rather than the real distance ρ. This is particularly useful since F(p) can be plotted versus $|p|$ with b as a parameter, thereby automatically including the dependence upon frequency, ground constants and distance. The remarks to follow will be concerned with the behavior of F(p) with regard to p. It is sufficient to consider only this special case since the behavior of F(w) with respect to w follows in exactly the same manner.

From (5), the complete range of ϕ_s and the corresponding ranges of b are composed of three parts which are indicated below:

Capacitive	Homogeneous	Highly Inductive
$-\pi/2 \leq \phi_s < 0$	$0 \leq \phi_s < \pi/4$	$\pi/4 \leq \phi_s \leq \pi/2$
$-3/2\pi \leq b < -\pi/2$	$-\pi/2 \leq b < 0$	$0 \leq b \leq \pi/2$

Wait [1957; 1962a,b; 1964] has expanded F(p) into two semiconvergent series for large $|p|$;

$$F(p) = -\frac{1}{2p} - \frac{1\cdot 3}{(2p)^2} - \frac{1\cdot 3\cdot 5}{(2p)^3} - \ldots, \qquad (7)$$

for $-2\pi < b < o$, and

$$F(p) = -2i\,(\pi p)^{1/2} e^{-p} - \frac{1}{2p} - \frac{1\cdot 3}{(2p)^2} - \frac{1\cdot 3\cdot 5}{(2p)^3} - \ldots, \qquad (8)$$

for $o < b < 2\pi$.

The first term in (8) corresponds to a "trapped" wave which is not present over a homogeneous or capacitive surface. The rest of the series is the usual asymptotic expansion of the Norton wave which exists over any type of flat surface.

Some 35 years ago, B. Rolf [1930] published a set of graphs which supposedly applied to the homogeneous earth where $-\pi/2 \leq b \leq o$. Unfortunately, he used an earlier derivation of F(p) due to Sommerfeld [1909] which had a sign error that was finally pointed out by Norton [1935]. Simply stated, this error was equivalent to interchanging b and -b. In other words, the complex conjugate of p was used in place of the proper quantity. Therefore, Rolf's calculations actually apply for a highly inductive earth instead of a homogeneous earth as he had thought. His curves displayed deep minima which he thought explained the phenomenon of fading in radio signals over a homogeneous earth. Actually, these variations result from the interference of the surface wave and one or more modes which propagate in the earth-ionosphere waveguide. In the case of an inductive earth, the variations result from the interference of the Norton surface wave and the trapped wave which have different phase velocities.

Although Rolf used a "semi-convergent" series for large $|p|$, the curves he obtained appear to be identical with those obtained here using the exact expression for F(p) and a computer. The "semi-convergent" series of Rolf does not converge except at $|p| = \infty$, but for sufficiently large $|p|$, the first few terms give a reasonable result provided too many terms are not used, hence the name "semi-convergent." Morse and Feshback [1953] discuss the nature of such expansions, and point out the precautions necessary before they can be applied with confidence. It will subsequently be shown that only the first two terms are sufficient to give a first approximation to F(p) for $|p| >> 1$. Wait [1962a, 1962b, 1964; Wait and Fraser, 1954] presented curves of $|F(p)|$ versus $|p|$ plotted on a linear

scale. Since the unusual behavior of $|F(p)|$ occurs for $|p| > 10$ where $|F(p)| << 1$, the deep minima did not appear in these curves.

More recently, L. E. Vogler [1964] pointed out that F(p) is not a smoothly varying function but can have "jogs" of considerable variation when $b > o$. His calculations were made from a power series expansion of (2), and were plotted on a logarithmic scale which clearly depicts the unsuspected behavior of F(p) for large $|p|$. Furthermore, he found that the phase of F(p) behaves in quite an unpredictable manner.

The following argument will tend to explain these unusual effects and also provide a means of predicting the approximate position of these irregularities as well as their amplitude and phase. This can be done quite simply with the aid of the first two terms of (8) which can be written

$$F(p) = 2\,|\pi p|^{1/2} e^{-|p|\cos b}\, e^{-i\left(\frac{\pi-b}{2} + |p|\sin b\right)} + \frac{e^{-i(\pi+b)}}{2|p|} \tag{9}$$

for $o \leq b \leq \pi/2$. These two terms are represented as phasors of the variable $|p|$ in figure 1. If b is held constant and $|p|$ is allowed to increase, the first term ("the trapped wave") which is changing in magnitude rotates clockwise about the phasor corresponding to the second term (the "Norton" surface wave) which is decreasing in magnitude but of constant phase $-(\pi+b)$. This rotation of the first term about the second leads to minima and maxima of the sum, i.e., if the magnitude of both phasors are not changing too rapidly, minima occur when

$$\frac{b-\pi}{2} - |p_n| \sin b = -(\pi+b) - (2n+1)\pi,$$

or $$|p_n| = \frac{3\,b + (4n+3)\pi}{2\sin b},\quad n = 0, 1, 2, ---, \tag{10}$$

and maxima occur when

$$\frac{b - \pi}{2} - |p_m| \sin b = -(\pi+b)-2m\pi,$$

$$\text{or} \quad |p_m| = \frac{3b + (4m + 1)\pi}{2 \sin b}, \ m = 0, 1, 2, ---, \tag{11}$$

provided $b \neq 0$. These results agree with those obtained by Vogler [1964] with the exception that m or n can be zero. However, when m or n are zero, (10) or (11) lead to small values of $|p|$ where (9) is not expected to be valid anyway.

Having located the minima and maxima of F(p), it is a simple matter to determine the approximate values of F(p) at these points. For example, at the nth minimum,

$$F(p) = \left[2|\pi p_n|^{1/2} e^{-|p_n|\cos b} - \frac{1}{2|p_n|} \right] e^{-i[(2n + 2)\pi + b]}, \tag{12}$$

and at the mth maximum,

$$F(p) = \left[2|\pi p_m|^{1/2} e^{-|p_m|\cos b} + \frac{1}{2|p_m|} \right] e^{-i[(2m + 1)\pi + b]}. \tag{13}$$

The magnitudes of (12) and (13) hold for all m and n, but the phases hold only for m and n less than some integer M yet to be specified.

The asymptotic values of arg $F(p)=-\Phi(p)$ can be easily found from (13) by allowing $|p|$ to increase until the first term is smaller in magnitude than the second term. The asymptotic value of $\Phi(p)$ is then $[(2M + 1)\pi + b]$ where M is the number of times the first term is larger than the second at the minima. In other words, $\Phi(p)$

increases by 2π everytime the trapped wave phasor extends into the fourth quadrant in figure 1 for increasing $|p|$.

Equation (9) was used to hand calculate the approximate values for b = 30°, 45°, 70°, and 90°. The results are compared in figures 2 and 3 with exact values as calculated from (2) by means of a digital computer. In order to assess the accuracy of (10) through (13), the values found from these equations are circled. It is apparent that, at least to a first order, (10) and (11) can be used to predict the locations of the maxima and minima of F(p), while (12) and (13) can be used to predict F(p) in both amplitude and phase. In figure 2 there is some perceptable error in the locations of the maxima and minima using (9) since the two phasors in figure 1 are simultaneously changing magnitude while the first is rotating about the second.

Although the amplitude of these "jogs" appears to be quite pronounced in figure 2, they are somewhat exaggerated due to a logarithmic scale used to plot $|F(p)|$. Even though these unusual variations occur where the field is small, they can cause extreme variations in the observed field. For example, for b = 70°, the vertical electric field varies about 20 dB between $|p|$ = 17 and $|p|$ = 20! Of course, such extreme changes require experimental verification, but they can possibly be used to explain pronounced variations observed for small changes in distance or frequency and certain types of highly inductive grounds.

Some of the maxima and minima in figure 2 are so small they are not evident. This happens when the magnitude of either term (9) is negligible in comparison to the other term. The results

of figures 2 and 3 indicate that (9) is adequate to calculate $F(p)$ when $|p| > 2$ and $b > 45°$. For lower values of b, $|p|$ must be progressively larger.

Figure 3 shows that for a very highly inductive surface where b is near 90°, $\Phi(p)$ is a rapidly increasing function with $|p|$ until the trapped wave term in (9) becomes less than the Norton wave term in magnitude, after which the phase oscillates about its asymptotic value and finally becomes constant. Furthermore, the asymptotic phase can vary considerably from one value of b to the next. For example, for $b = 65°$, $M = 1$ and the asymptotic phase of $F(p)$ is $-605°$, while for $b = 66°$, $M = 2$ and the asymptotic phase is $-966°$. The ability of the phasor approach to predict these discontinuities of 2π and the asymptotic values should prove to be of utility.

It should be noted that $\Phi(p)$ is not uniquely defined over a highly inductive earth unless the method of evaluating it is stated. Here $\Phi(p)$ has been calculated by fixing b and increasing $|p|$, but it is just as reasonable to fix $|p|$ and increase b when calculating $\Phi(p)$. Through an academic peculiarity, these two methods yield different values for $\Phi(p)$ when $b > 51°$, and these different values must be interpreted in different ways, i.e., they actually represent different quantities. When $\Phi(p)$ is calculated by fixing b and increasing $|p|$, the $\Phi(p)$ is being summed over the entire path traversed by the ground wave. When a small change in b results in a 2π difference in the asymptotic values of $\Phi(p)$, the distance from transmitter to receiver is one wavelength longer for the larger b value. If $\Phi(p)$ is calculated by fixing $|p|$ and increasing b,

$\Phi(p)$ represents the relative phase change in the signal received at a fixed distance from the transmitter as the angle ϕ_s increases. As b changes, the phase of the received signal also changes, but the change is continuous; there are no 2π jumps. When $|p|=\infty$, $\Phi(p)$ approaches an asymptotic value of $(\pi+b)$ for all $b < 90°$. Thus, it is necessary to clarify whether $\Phi(p)$ is being considered in a cumulative sense or a relative sense before a unique set of values is obtained, the two asymptotic values being different by $2M\pi$.

To further illustrate this non-uniqueness, take the limit of (8) as $|p|\to\infty$. Then the result is

$$\lim_{|p|\to\infty} F(p) = \frac{-1}{2p} = \frac{e^{-i(\pi+b)}}{2\ |p|} ,$$

and the non-cumulative asymptotic value of $\Phi(p)$ is $(\pi+b)$ which agrees with Wait's results [Wait and Fraser, 1954; Wait, 1962a, b, 1964].

Finally, it is apparent from these results that the phase velocities of the trapped wave and the Norton surface wave are both less than c, the speed of light. Such waves are said to be "slow waves", and are non-radiating (bound waves). Furthermore, their resultant is also a slow wave. This is most easily seen by plotting the familiar dispersion diagram of k versus β, where β is the phase constant of the wave determined by $\exp[-i(kd+\Phi)] = \exp(-i\beta d)$

or $$\beta = k\left(1 + \frac{|\Delta|^2}{2}\,\frac{\Phi(p)}{|p|}\right) .$$

Thus, the phase velocity is given by

$$v_{ph} \equiv \frac{\omega}{\beta} = \frac{c}{1 + \frac{|\Delta|^2}{2}\frac{\Phi(p)}{|p|}}, \text{ and } v_{ph} < c \text{ since } \Phi(p) > 0.$$

The group velocity is given by $v_g \equiv d\omega/d\beta$ which represents the slope of the dispersion curve. Since the phase $\Phi(p)$ displays maxima and minima, it is possible that v_g may be positive, negative or zero and may in fact exceed the speed of light, c. Such a situation arises in a region of considerable dispersion where $\Phi(p)$ displays very rapid changes as $|p|$ varies. The group velocity should not be confused with "signal" velocity or velocity of energy transport which are decidedly different in a region of high dispersion. The signal and energy transport velocities are always less than c, and become somewhat arbitrary and difficult to evaluate in a region of high dispersion. For a normally dispersive region, they are very nearly equal to the group velocity [Brillouin, 1960]. Furthermore, the envelope of any modulated signal in a highly dispersive region will be altered considerably upon passing through this region.

3. Calculations

As already pointed out, Wait [1962a, 1962b, 1964; Wait and Fraser, 1954] has given curves of F(p) versus $|p|$ with b as a parameter for $o \leq b \leq \pi/2$. However, these graphs were plotted on a linear scale and do not show the unusual behavior of F(p) for large $|p|$. Rolf's [1930] curves were intended to apply to $-\pi/2 \leq b \leq o$, however his curves are actually valid for $o \leq b \leq \pi/2$, the case which is being considered here. These curves have other disadvantages besides this obvious factor of confusion. First,

they are plotted in an unusual nomograph form which requires the knowledge of the effective dielectric constant and conductivity in electrostatic units rather than the almost universally accepted "numerical distance." Hence, considerable conversion is necessary before the graphs become useful. Second, phase data has been completely omitted. This data is particularly important for certain types of modern navigation systems at LF and VLF.

The case where $-\pi \leq b \leq o$ has already been treated adequately by Norton [1936, 1941; King, 1956] who gives graphs for b in this range. They are sufficiently detailed and properly presented to be useful to practicing radio engineers.

However, as mentioned by Vogler [1964], there seems to be a lack of calculations for $o \leq b \leq \pi/2$ corresponding to a highly inductive surface. To this end, extensive calculations have been made on a digital computer for F(p) using the exact form given by (3), and the results are shown on figures 4 and 5. Less complete curves have been given by Biggs and Swarm [1965].

If the exact evaluation of the attenuation function is desired in detail for a "b" value not given in figures 4 and 5, then it can be found using an identity due to Wait [1965a]. F(p) can be simply calculated for $o \leq b$ using existing data for $b \leq o$. Using (2), the following identity is easily proven;

$$F(p) = -2i(\pi p)^{1/2} e^{-p} + F^{*}(p^{*}), \tag{14}$$

where the asterisk denotes the complex conjugate. If, however, an approximate evaluation of F(p) is sufficient, then (9) yields good results, particularly for large values of $|p|$ where the unusual features occur.

4. Conclusions

The ground wave attenuation function, F(p) for propagation over a passive highly inductive flat surface can be simply calculated for large numerical distances, p. The unusual behavior of F(p) over such a surface is due to the interaction of the "trapped" and the Norton surface waves. One of the dominant characteristics of the trapped wave (which exists only over an inductive surface) is its rapid phase variation compared to the slow variation of the Norton wave. When the complex phasors representing these two waves are added, deep minima in $|F(p)|$ and large phase variations of the resultant phasor can occur. A simple method is given for locating the approximate values of $|p|$ where the maxima and minima of $|F(p)|$ occur, as well as a method for determining F(p) at these points. The asymptotic cumulative phase of F(p) for large $|p|$ displays discrete steps of 2π for certain changes in arg p. The phasor approach also proves useful in predicting this asymptotic phase of F(p).

Detailed graphs of $|F(p)|$ and arg F(p) versus $|p|$ with arg p as a parameter are given. Furthermore, a simple formula due to Wait has been given which permits evaluation of F(p) for arg p $\geq$ o using published curves of F(p) for arg p $\leq$ o.

With these tools, a quick analysis of the unusual behavior of F(p) is now possible, whereas a long tedious evaluation was necessary previously.

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FIGURE CAPTIONS

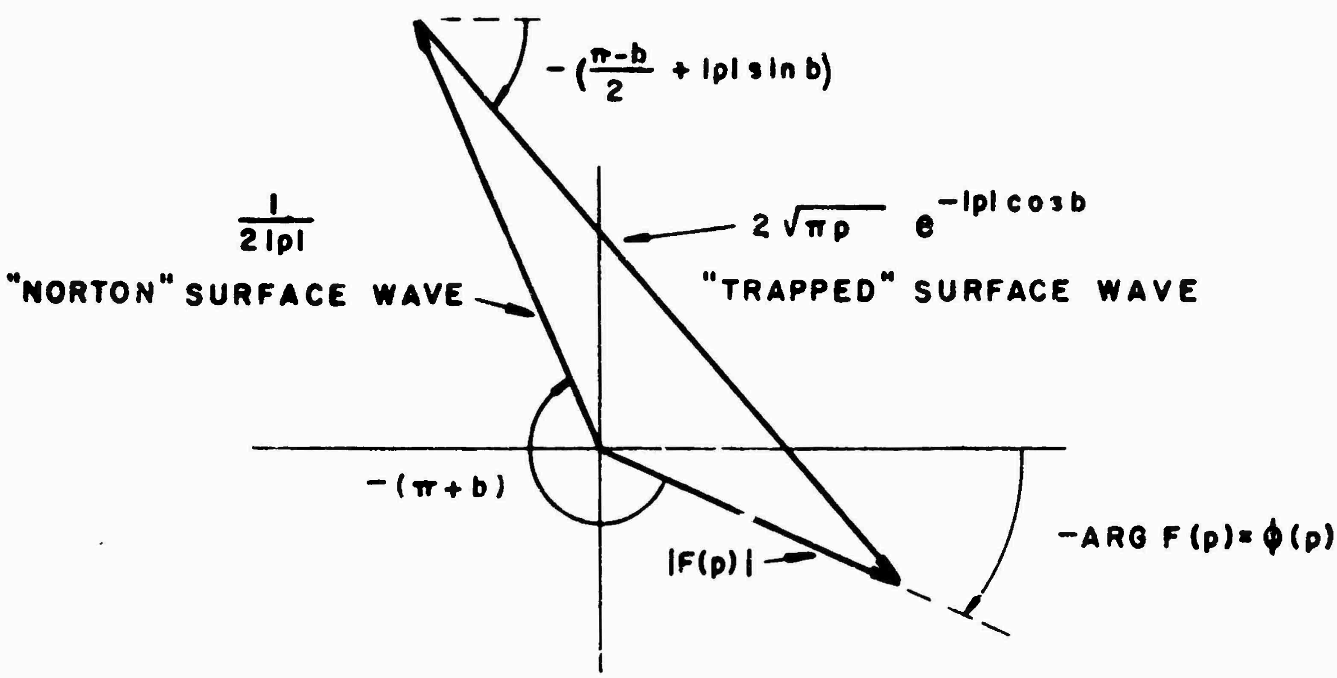
Figure 1. Phasor diagram representing the interaction of the Norton surface wave and the trapped wave. The sum of these two waves represents the total groundwave which is characterized by F(p).

Figure 2. Magnitude of the groundwave attenuation function versus the magnitude of the numerical distance, p. The parameter is b, the argument of the numerical distance. The exact calculations were obtained using Eq. (2), and the approximate calculations were obtained using only the first two terms of Eq. (8).

Figure 3. Phase lag of the groundwave attenuation function versus the magnitude of the numerical distance, p. The parameter is b, the argument of the numerical distance. The exact calculations were obtained using Eq. (2), and the approximate calculations were obtained using only the first two terms of Eq. (8).

Figure 4. Parametric curves of the magnitude of the groundwave attenuation function F(p) versus the magnitude of the numerical distance, p. $p = |p|e^{ib}$.

Figure 5. Parametric curves of the phase lag of the groundwave attenuation function $\Phi(p)$ versus the magnitude of the numerical distance p. $p = |p|\ e^{ib}$ and $F(p) = |F(p)|e^{-i\Phi(p)}$.



$-\left(\frac{\pi-b}{2}+|p|\sin b\right)$
$\frac{1}{2|p|}$
"NORTON" SURFACE WAVE
$2\sqrt{\pi p}\ e^{-|p|\cos b}$
"TRAPPED" SURFACE WAVE
$-(\pi+b)$
$|F(p)|$
$-\text{ARG } F(p)=\phi(p)$

Fig. 1

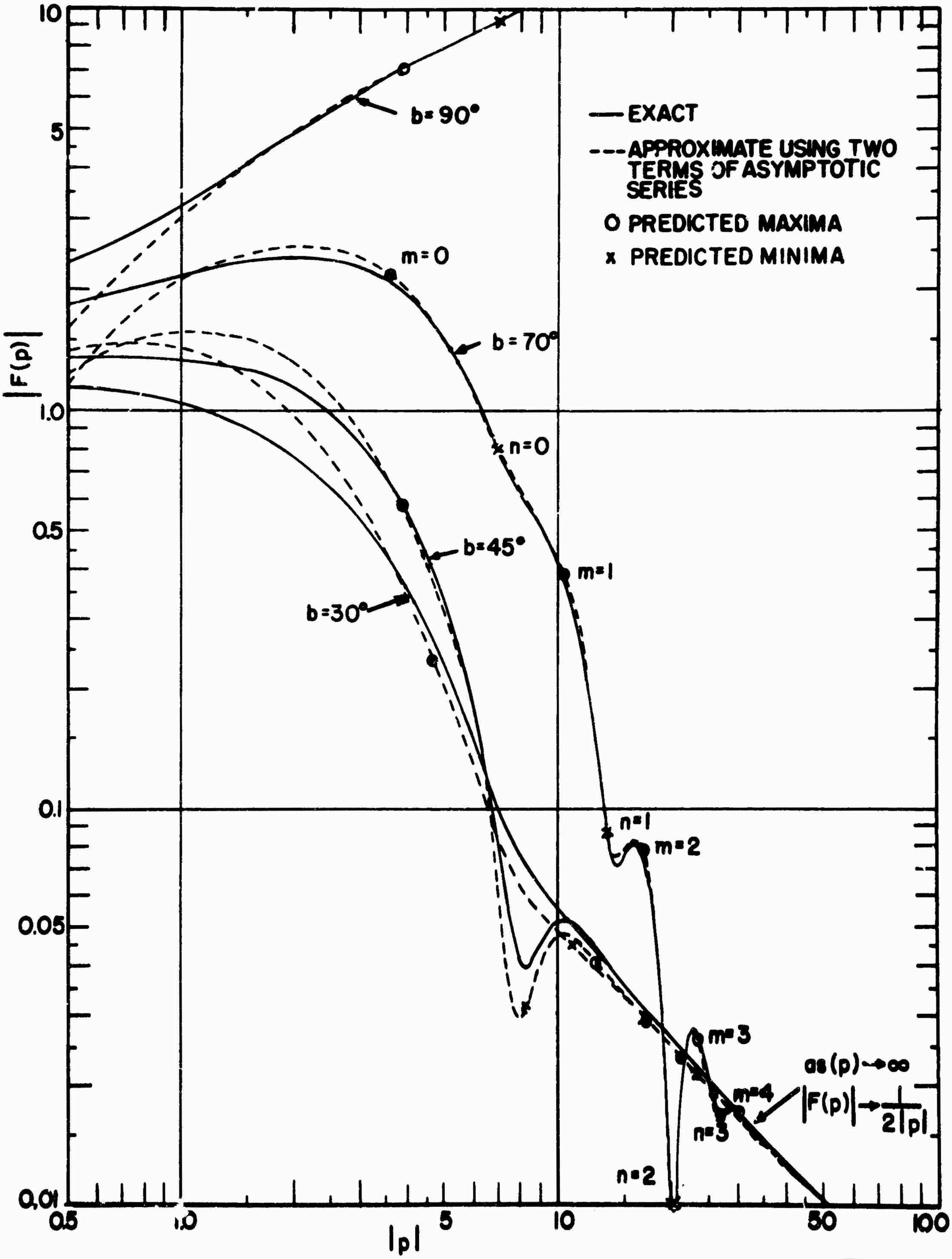
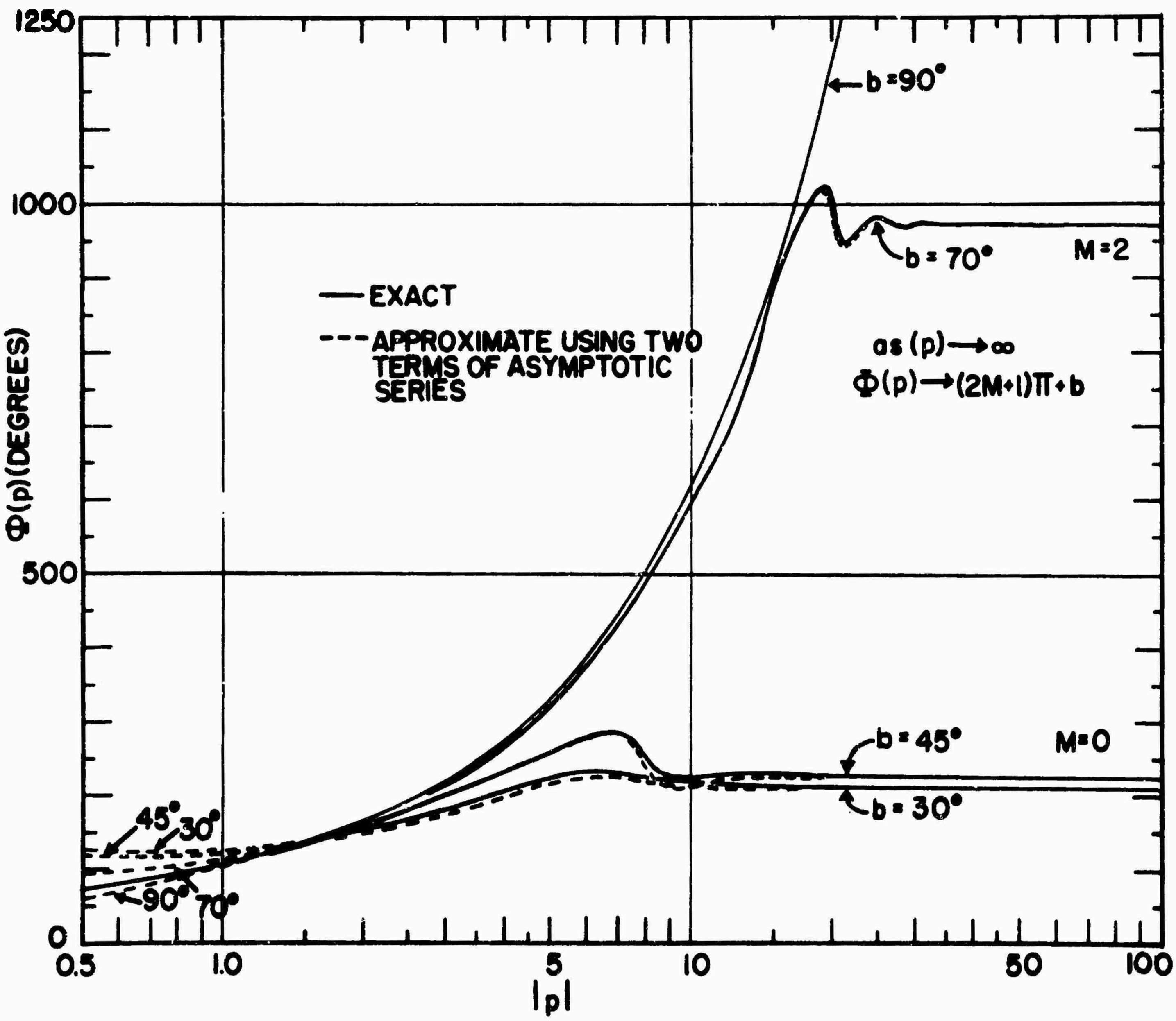
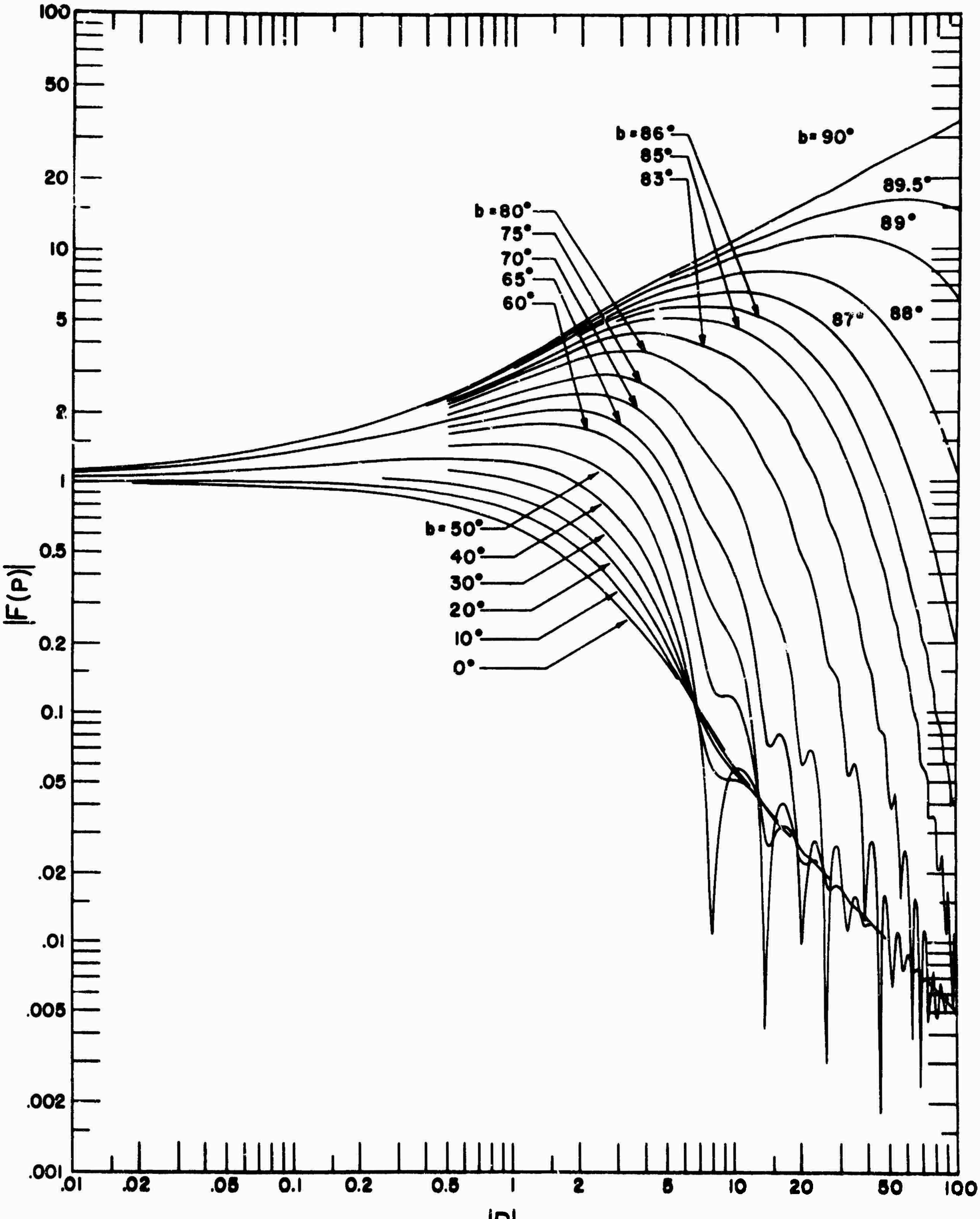

EXACT
APPROXIMATE USING TWO TERMS OF ASYMPTOTIC SERIES
O PREDICTED MAXIMA
x PREDICTED MINIMA
b=90°
b=70°
b=45°
b=30°
m=0
m=1
m=2
m=3
m=4
n=0
n=1
n=2
n=3
as |p|→∞
|F(p)|→1/(2|p|)
|F(p)|
|p|
10
5
1.0
0.5
0.1
0.05
0.01
0.5
1.0
5
10
50
100

Fig. 2



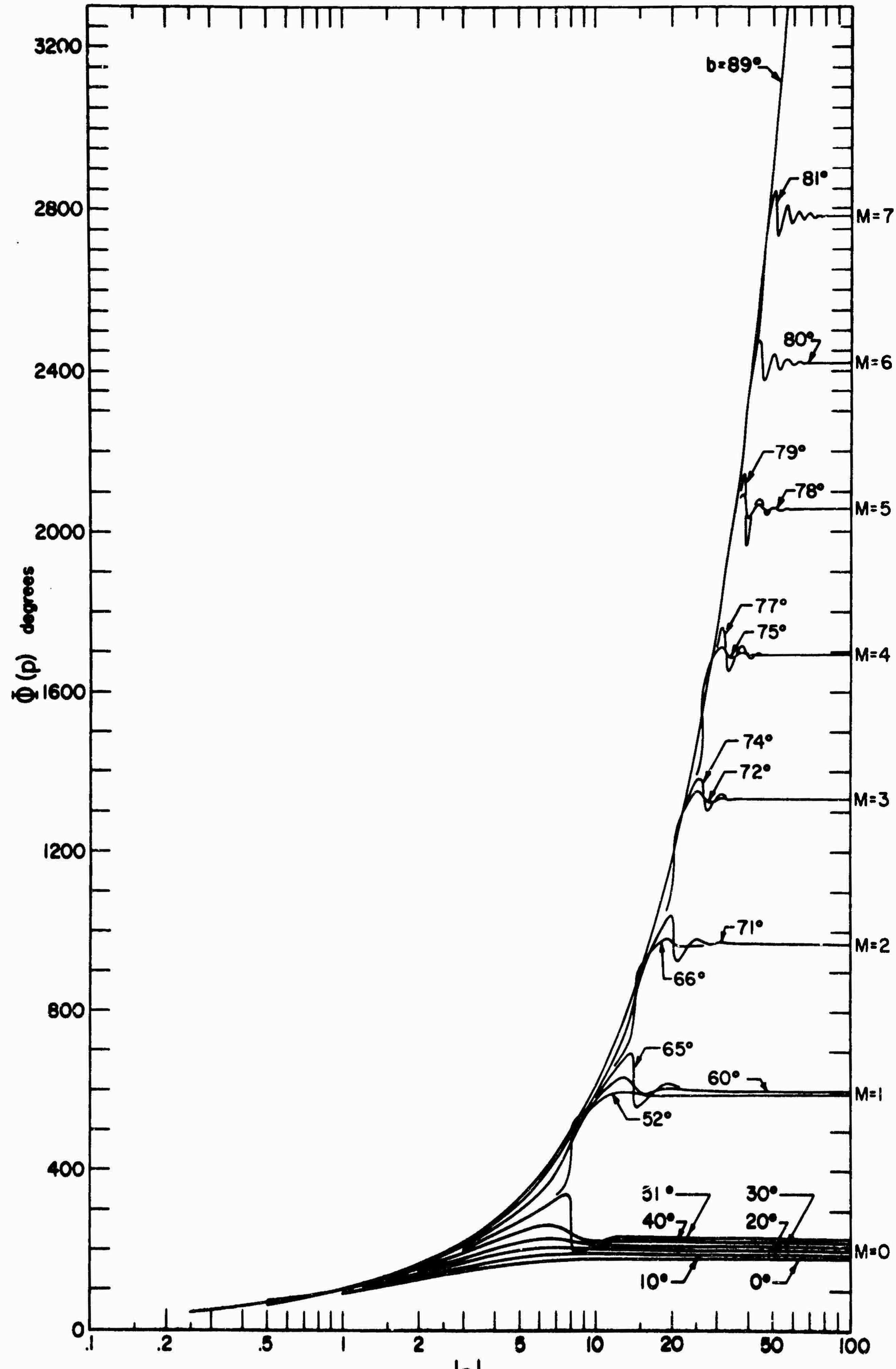
1250
1000
500
0
0.5
1.0
5
10
50
100
|p|
Φ(p)(DEGREES)
b = 90°
b = 70°
M = 2
EXACT
APPROXIMATE USING TWO TERMS OF ASYMPTOTIC SERIES
as (p) → ∞
Φ(p) → (2M+1)π + b
b = 45°
M = 0
b = 30°
45°
30°
90°
70°

Fig. 3



100
50
20
10
5
2
1
0.5
0.2
0.1
.05
.02
.01
.005
.002
.001
|F(P)|
.01
.02
.05
0.1
0.2
0.5
1
2
5
10
20
50
100
|P|
b = 86°
85°
83°
b = 90°
89.5°
89°
88°
87°
b = 80°
75°
70°
65°
60°
b = 50°
40°
30°
20°
10°
0°

Fig. 4



b=89°
81°
M=7
80°
M=6
79°
78°
M=5
77°
75°
M=4
74°
72°
M=3
71°
M=2
66°
65°
60°
M=1
52°
51°
40°
30°
20°
M=0
10°
0°
3200
2800
2400
2000
1600
1200
800
400
0
$\bar{\phi}$(p) degrees
.1
.2
.5
1
2
5
10
20
50
100
|p|

Fig. 5

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13. ABSTRACT

Propagation of an electromagnetic ground wave over a plane surface in which the argument of the surface impedance is greater than $\pi/4$ but less than $\pi/2$ is considered in some detail. The numerical distance, p, over such a surface is characterized by $0 \leq \arg p \leq \pi/2$. The ground wave behaves in a rather unusual manner, and this is attributed to the interaction of phasors representing a trapped wave and a Norton surface. Approximate expressions are derived which determine the magnitude of the ground wave attenuation function at its maxima and minima as well as the phase at these points and the numerical distances where these maxima and minima occur. A method is also given for estimating the asymptotic phase for large $|p|$ which was previously not possible. Finally, detailed curves are presented which show the amplitude and phase of the ground wave attenuation function versus p. These curves should prove useful to practicing radio engineers attempting to make calculations for surface wave propagation over corrugated, stratified or rough surfaces.

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	ROLE	WT	ROLE	WT	ROLE	WT
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